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Roll. No:

NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

B.Tech

SEM: IV - THEORY EXAMINATION (2021 - 2022)

Subject: Engineering Mathematics-III

Time: 3 Hours Max. Marks: 100

General Instructions:

1. The question paper comprises three sections, A, B, and C. You are expected to answer them as directed.
2. Section A - Question No- 1 is 1 mark each & Question No- 2 carries 2 mark each.
3. Section B - Question No-3 is based on external choice carrying 6 marks each.
4. Section C - Questions No. 4-8 are within unit choice questions carrying 10 marks each.
5. No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.

SECTION A 20

1. Attempt all parts:-

1-a. $\lim_{z\to 0}\frac{z}{\bar{z}}$ (CO1) 1

(a) Limit exists

(b) Limit does not exist

(c) Limit exists and equal to 1

(d) None of these

1-b. The conjugate harmonic of the function $u = x^2 - y^2 - y$ is (CO1) 1

(a) $2xy + 2$

(b) $2xy + x$

(c) $-4xy + y$

(d) None of these

1-c. If there is no pole inside and on the contour, then the value of integral is (CO2) 1

(a) ∞

(b) 0

(c) -1

(d) None of these

1-d. The singular points of $f(z) = \frac{1}{z(z-1)^2}$ are (CO2) 1

(a) 0,1,-1

(b) 0,1,1

(c) 1,-1

(d) None of these

1-e. Which of the following is a two-dimensional wave equation? (CO3) 1

(a) $\frac{\partial u}{\partial t} = c^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$

(b) $u = c^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$

(c) $\frac{\partial^2 u}{\partial t^2} = c^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$

(d) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

1-f. Classification of P.D.E. $5u_{xx}-9u_{xt}+4u_{tt}=0$ is: (CO3) 1

(a) Hyperbolic
(b) Elliptic
(c) parabolic
(d) None of these

1-g. Find the Fourier transform of an exponential function $F(t) = e^{-at}u(t)$, $a > 0$. (CO4) 1

(a) $\frac{1}{(a+ip)}$
(b) $\frac{1}{(a-ip)}$
(c) $\frac{1}{(ip-a)}$
(d) $\frac{1}{(-ip-a)}$

1-h. $Z^{-1}\left(\frac{z}{z+3}\right)$ is (CO4) 1

(a) $(3)^k$
(b) $(-3)^k$
(c) $(3)^{-k}$
(d) None of these

1-i. A boat goes 8 km in one hour along the stream and 2 km in one hour against the stream. The speed in km/hr of the stream is ? (CO5) 1

(a) 2
(b) 3
(c) 4
(d) 5

1-j. The calendar for the year 2007 will be the same for the year: (CO5) 1

(a) 2014
(b) 2017
(c) 2018
(d) None of these

2. Attempt all parts:-

2.a. Write the necessary and sufficient condition for $f(z)$ to be analytic. (CO1) 2

2.b. Evaluate: $\oint_C \frac{e^z}{z^2+1} dz$; $C \equiv |z| = \frac{3}{2}$. (CO2) 2

2.c. Solve the PDE: $(D^3 - D^2D' - DD'^2 - D'^3)z = 0$. (CO3) 2

2.d. Find $F_s^{-1}\left(\frac{1}{p}\right)$. (CO4) 2

2.e. A, P, R, X, S and Z are sitting in a row. S and Z are in the Centre. A and P are at the ends. R is sitting to the left of A. Who is to the right of P? (CO5) 2

SECTION B 30

3. Answer any five of the following:-

3-a. Discuss the analyticity of the function $f(z) = Re(z^3)$ in the complex plane . (CO1) 6

3-b. Find regular function whose imaginary part is $\frac{x}{x^2+y^2} + \cosh x \cos y$. (CO1) 6

3-c. Expand $f(z) = \frac{z}{(z-1)(2-z)}$ in Laurent series valid for the regions (CO2) 6

(a) $|z-1| > 1$ (b) $0 < |z-2| < 1$.

3-d. Discuss the nature of singularity of $f(z) = \frac{e^{1/z}}{z^2}$. (CO2) 6

3.e. Use the method of separation of variables to solve the equation $\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}$, subject to condition $u(0,y) = 8e^{-3y}$. (CO3) 6

3.f. Find the $Z-$transform of $f(k) = \cos(\alpha k), k \geq 0$. (CO4) 6

3.g. Two pipes A and B can fill a tank in 12 minutes and 15 minutes respectively while a third pipe C can empty the full tank in 20 minutes. All the three pipes are opened in the beginning. However, pipe C is closed 6 minutes before the tank is filled. In what time will the tank be full? (CO5) 6

SECTION C 50

4. Answer any one of the following:-

4-a. Show that for the function $f(z) = \left\{ \frac{2xy(x+iy)}{x^2+y^2}, \quad z \neq 0 \right\}, f(0) = 0, z = 0$ 10

the C-R equations are satisfied at origin but derivatives of f(z) does not exist at origin. (CO1)

4-b. Find the bilinear transformation which maps the points $z = 1, i, -1$ into the points $w = i, 0, -i$. 10

Hence find the image of $|z| < 1$. (CO1)

5. Answer any one of the following:-

5-a. Verify Cauchy integral theorem for $f(z) = e^{iz}$ along the boundary of the triangle with the vertices $1+i, -1+i$ and $-1-i$. (CO2) 10

5-b. Evaluate $\int_0^{2\pi} \frac{1}{5+4\cos\theta} d\theta$ using contour integration. (CO2) 10

6. Answer any one of the following:-

6-a. Solve the PDE: $(D^2 - D'^2 - 3D + 3D')z = xy + e^{x+2y}$. (CO3) 10

6-b. Find the temperature of the bar of length 2 whose ends are kept at zero and lateral surface insulated if the initial temperature is sin (πx/2)+ 3sin(5πx/2) (CO3) 10

7. Answer any one of the following:-

7-a. Solve by $z-$transform: $y_{k+2} + 4y_{k+1} + 3y_k = 3^k; y_0 = 0, y_1 = 1$. (CO4) 10

7-b. Use Fourier sine transform to solve the equation $\frac{\partial u}{\partial t} = 2\frac{\partial^2 u}{\partial x^2}$ under the conditions: (CO4) 10

(i) $u(0, t) = 0$

(ii) $u(x, 0) = e^{-x}$

(iii) $u(x, t)$ is bounded.

8. Answer any one of the following:-

8-a. (i) A and B can together finish a work in 30 days. They worked together for 20 days and then B left. After another 20 days, A finished the remaining work. In how many days A alone can finish the job ? 10

(ii) A man and a boy together can do a certain amount of digging in 40 days. Their speeds in digging are in the ratio of 8 : 5. How many days will the boy take to complete the work if engaged alone? (CO5)

8-b. (i) Two trains of lengths 120 m and 90 m are running with speeds of 80 km/hr and 55 km/hr respectively towards each other on parallel lines. If they are 90 m apart, after how many seconds they will cross each other? 10

(ii) A car travels from P to Q at a constant speed. If its speed were increased by 10 km/hr, it would have taken one hour lesser to cover the distance. It would have taken further 45 minutes lesser if the speed was further increased by 10 km/hr. What is the distance between the two cities? (CO5)